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Abstract—A time-interleaved sigma–delta modulator using the output prediction scheme is proposed. This approach uses only one integrator channel along with incomplete integrator output terms to eliminate the quantizer domino which is a key limit for practical circuit implementation of conventional time-interleaved sigma–delta modulators. In addition, channel mismatch effects due to mismatch within multiple integrator feedback paths can be reduced by optimizing the feedback path. An equivalent two-channel time-interleaved version of the conventional second-order sigma–delta modulator is realized to verify the proposed method.

Index Terms—Channel mismatch, incomplete integrator outputs, output prediction, quantizer domino, sigma–delta modulators, time-interleaved.

I. INTRODUCTION

SIGMA–DELTAS (ΣΔ) techniques have been widely used for low-bandwidth and high-resolution applications owing to the oversampling and noise shaping feature [1], [2]. However, ΣΔ applications are being extended to new telecommunication areas, which typically require megahertz-range signal bandwidth with a sampling clock near 100 MHz [3]. With high sampling clocks, the CMOS implementation of ΣΔ modulators is problematic due to the high-frequency limitations of the opamps and sampling switches.

Time-interleaved (TI) ΣΔ modulators are an attractive solution for high-speed applications, since the oversampling rate (OSR) can be increased without speeding up the analog blocks [4]–[7]. However, the recursive operation of ΣΔ modulators, which is not used in Nyquist rate interleaved converters [8], complicates the direct conversion into their equivalent TI structures [6]. Among the TI concepts for ΣΔ modulators, the block digital filtering [4], [5] and the extended hardware reduction approach [6], [7] conceptually works well, but quantizer domino and channel mismatch effects limit the circuit implementation. Quantizer domino occurs when a certain quantizer output is connected to another quantizer input via an analog block without passing through a delay element. This situation is unavoidable in M-channel TI ΣΔ modulators, where M-1 consecutive time slot outputs of the Lth integrator can be written as

$$p_L(n+j) = p_{zi}(n+j) + p_{rzi}(n+j) + p_{yzi}(n+j),$$

for $j > 0$ (1)

where $n$ and $j$ are integers. Here $p_{zi}$ is the zero input response or natural response, and $p_{rzi}$ and $p_{yzi}$ are the zero state response or forced response of the subsystem with respect to the input $x$ and the feedback $y$. Assuming $p_L(n)$ is the initial output, the $M-1$ consecutive time slot outputs of the $L$th integrator can be written as

$$p_L(n+j) = p_L(n) + \sum_{i=1}^{L-1} h_i(j)p_L(n) + \sum_{i=0}^{j-1} s(j-i)x(n+i)$$

$$- \sum_{i=0}^{j-1} f(j-i)y(n+i),$$

for $j = 1, \ldots, M-1$ (2)

where the first two terms correspond to the zero input response, and the remaining two terms represent the zero state response with $x$ and $y$ applied from time slot $n$. In addition, $h_i$ is a coefficient due to the recursive operation of the $i$th time slot integrator outputs $p_i(n)$ for $i = 1, \ldots, L$. Moreover, $s$ and $f$ are the...
impulse response determined at the $L$th integrator by the input $x$ and feedback $y_i$, respectively.

Since the TI operation requires the future outputs of $p_{L,i}$, it is convenient to redefine each variable used in (2) such that it is assigned to the $j$th channel of the TI modulator as

$$x_j(n') = x[Mr + j], \quad \text{for } j = 1, \ldots, M \quad (3a)$$

$$y_j(n') = y[Mr + j], \quad \text{for } j = 1, \ldots, M \quad (3b)$$

$$p_{i1}(n') = p_i[Mr + 1], \quad \text{for } i = 1, \ldots, L \quad (3c)$$

where $r$ is an integer for $r > 0$, $n'$ is the decimated timing of the TI modulator, and $p_{i1}(n')$ is the zero input initial condition of the $i$th integrator, which corresponds to the $n'$th time slot integrator output. Using (3a)–(3c), (2) can be rewritten as

$$p_{L,j}(n') = p_{L,1}(n') + \sum_{i=1}^{j-1} h_i(j-1)p_{i1}(n')$$

$$+ \sum_{i=1}^{j-1} s(j-i)x_i(n') - \sum_{i=1}^{j-1} f(j-i)y_i(n'), \quad \text{for } j = 2, \ldots, M \quad (4)$$

To generate the $p_{L,j}$ terms, it is necessary to know the future signals $x[Mr + j]$ and $y[Mr + j]$ for $j = 2, \ldots, M$. For the inputs $x$, using an $M$-clock sample-and-hold version $x_{sh}$ instead of $x$ permits the lack of the future inputs: $x_{sh}[Mr + j] = x[Mr]$. Alternatively, the future values of $x$ can be determined by using an $(M-1)$-clock delayed version of $x$, that is $x_{dl}[Mr + j] = x[Mr + j]$. Unfortunately, using the future feedback terms $y[Mr + j]$ for $j > 1$ directly leads to quantizer domino—only $y_1(n')$ can be determined by quantizing the initial condition $p_{L,1}(n')$. Since it is not possible to generate the future terms of $p_{L,j}(n')$ for $j > 1$, let us consider the incomplete integrator outputs $p_{L,j}(n')$, defined as

$$p_{L,j}(n') = p_{L,1}(n') + \sum_{i=1}^{j-1} h_i(j-1)p_{i1}(n')$$

$$+ \sum_{i=1}^{j-1} s(j-i)x_i(n'), \quad \text{for } j = 2, \ldots, M \quad (5)$$

Furthermore, using (4), (5) can be rewritten as

$$p_{L,j}(n') = p_{L,j}(n') + \sum_{i=1}^{j-1} f(j-i)y_i(n'), \quad \text{for } j = 2, \ldots, M \quad (6)$$

Since the quantization of $p_{L,j}(n')$ leads to the modulator output $y_j(n')$, the quantization of (6) yields

$$y_j(n') + \varepsilon_j(n') = y_j(n') + \varepsilon_j(n') + \sum_{i=1}^{j-1} f(j-i)y_i(n'), \quad \text{for } j = 2, \ldots, M \quad (7)$$

where $\varepsilon_j(n')$ is the quantization error which affects the SNR of the TI $\Sigma\Delta$ modulator by contributing to the quantization noise power. Finally, (7) leads to

$$y_j(n') = y_j(n') + \sum_{i=1}^{j-1} f(j-i)y_i(n'), \quad \text{for } j = 2, \ldots, M \quad (8)$$

Therefore, the complete output set $y_j(n')$ for $j = 1, \ldots, M$ can be obtained by a suitable digital processing of the complete output $y_1(n')$ and incomplete outputs $y_j(n')$ for $j = 2, \ldots, M$. Here, the required digital processing does not cause quantizer domino. However, the quantizers which generate the $y_j(n')$
terms require a larger number of quantization levels with respect to the conventional case, since the dynamic range of \( y_{j}^{2}(n') \) is larger than \( y_{j}(n') \). Fig. 1(b) represents the TI version of the \( M \)-channel \( L \)th order \( \Sigma \Delta \) modulator, where the input signal \( x \) is passed through a delay line before the decimation. The proposed scheme requires only one integrator channel which generates the zero input initial conditions \( p_{i1}(n') \) for \( i = 1, \ldots, L \). The subsystem enclosed within the dotted line corresponds to the linear section of Fig. 1(a), which directly implements (5). Also, \( d_{i} \) for \( i = 1, \ldots, L \) denote the feedback signal of the \( i \)th integrator, which is necessary for generating the zero input initial conditions.

B. Two-Channel TI Second-Order \( \Sigma \Delta \) Modulator

The proposed TI scheme is applied to a conventional second-order \( \Sigma \Delta \) modulator shown in Fig. 2(a), to realize a two-channel TI version which is shown in Fig. 2(b). Using (3a)–(3c), for \( M = 2 \) and \( j = 1, 2 \), leads to the zero input initial conditions

\[
p_{11}(n') = p_{11}(n' - 1) + 0.5[x_{2}(n' - 1) + x_{1}(n' - 1) - y_{2}(n' - 1) - y_{1}(n' - 1)]
\]

(9)

\[
p_{21}(n') = p_{21}(n' - 1) + 2[p_{21}(n' - 1) + 0.5x_{1}(n' - 1) - y_{2}(n' - 1) - 1.5y_{1}(n' - 1)]
\]

(10)

where \( y_{1}(n' - 1) \) and \( y_{2}(n' - 1) \) are the feedback terms. Here, \( p_{21}(n') \) is used for the input of \( Q_{1} \), which will be the complete modulator output \( y_{1}(n') \), after quantization. Furthermore, using (5), the input of \( Q_{2} \) can be expressed as

\[
p_{22}(n') = p_{21}(n') + 2p_{11}(n').
\]

(11)

Also, based on (6) and (8), the incomplete modulator output obtained by quantizing (11) is

\[
y_{2}^{2}(n') = y_{2}(n') + 2y_{1}(n').
\]

(12)

Therefore, the complete modulator output \( y_{2}(n') \) can be simply obtained by subtracting the \( 2y_{1}(n') \) term from (12).

In order to reduce the channel mismatch effect, the feedback of the first integrator is applied only through DAC1. By contrast, the feedback path of the second integrator is divided into two parts using the additional DAC2. Observe that the second integrator is less sensitive to mismatch errors, and moreover using a single feedback path would require a higher resolution for the DAC due to the \(-1.5y_{1}(n' - 1)\) term in (10). Thus, \( Q_{1} \) and DAC2 can have the same resolution as the conventional \( \Sigma \Delta \) modulator, whereas only an extra bit is required for \( Q_{2} \) and DAC1. Now the two-channel TI modulator shown in Fig. 2(b) is completely free of the quantizer domino. It can be directly converted into the circuit level without redistributing the delays as in [5]. Fig. 3 shows the SC implementation of the proposed two-channel TI modulator using three opamps. The output of each integrator is valid during clock phase 2, thus an additional opamp sums the integrator outputs during clock phase 2, and determines the incomplete integrator output \( p_{22}^{c} \) stated in (11). Since the outputs of \( Q_{1} \) and \( Q_{2} \) are latched at the falling edge of clock phase 2, DAC1 and DAC2 are able to generate their analog outputs during the next clock phase 1. The digital blocks \( S_{1}, S_{2}, \) and \( S_{3} \) realize the digital processing operation depicted in Fig. 2(b).

III. SIMULATION RESULTS

The performance of the proposed two-channel TI second-order \( \Sigma \Delta \) modulator is compared with the conventional single-channel and other two-channel TI structures through behavioral level simulations. A 2-b quantizer was used for the conventional second-order \( \Sigma \Delta \) modulator shown in Fig. 2(a).

Fig. 4 shows the output spectrum of the conventional second-order \( \Sigma \Delta \) modulator and the proposed two-channel TI version with \(-6\) dBFS, 152.59-kHz sinusoidal input. For both modulators, the internal clock frequency was set to \( f_{s} = 100 \) MHz, with a signal bandwidth of 1.56 MHz. Therefore, the effective clock frequency of the two-channel TI modulator is \( 2f_{s} \), doubling the effective OSR of the modulator. As expected, the SNDR improvement of the two-channel TI second-order \( \Sigma \Delta \) modulator was approximately 15 dB.
Channel mismatch effect is a potential drawback for multichannel TI \( \Sigma \Delta \) modulators. With channel mismatch, the in-band noise level of the \( M \)-channel TI \( \Sigma \Delta \) modulator will increase, since the spectral components around \( 2\pi i/M \) for \( i = 1, \ldots, M - 1 \) will be folded back into the signal band [5], [6]. Fig. 5 shows the output spectra of three different two-channel TI second-order \( \Sigma \Delta \) modulators with 0.5% channel mismatch. Here, TI-(I) refers to the block digital filtering scheme with the \( k \)-factor technique [5], and TI-(II) denotes the extended hardware simplified version [6]. Results show the SNDR degradation of the proposed TI modulator is around 2 dB with 0.5% channel mismatch, which is less than the other two-channel TI structures. This is mainly due to the single feedback path of the first integrator, since the feedback terms of the first integrator will be affected by an identical error, thus reducing the spur around \( f_s \). As expected, the SNDR degradation due to the mismatch between the two feedback paths of the second integrator was negligible.

**IV. CONCLUSION**

In this brief, we have presented a new TI scheme for \( \Sigma \Delta \) modulators. The proposed method is able to eliminate the quantizer domino within the TI \( \Sigma \Delta \) modulator, regardless of the channel count. The reduced channel mismatch effect is an extra benefit of the proposed approach which is enabled by using a single integrator channel with an optimized feedback path. The SNDR degradation of the proposed two-channel TI second-order \( \Sigma \Delta \) modulator respect to channel mismatch effect was relatively less compared to other two-channel TI \( \Sigma \Delta \) modulators. Finally, the proposed TI scheme can be even practical for circuit implementation of TI \( \Sigma \Delta \) modulators with more than two channels.
REFERENCES


