THE NOISE SPECTRUM
OF A DEVICE WITH BULK NEGATIVE DIFFERENTIAL MOBILITY

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Abstract
The noise properties of GaAs bulk diodes, biased with high electric field, have been calculated. We have considered the noise arising from velocity fluctuations, leading to Nyquist noise in thermal equilibrium. It has been calculated the noise voltage power spectrum at low frequencies as a function of the bias field and of the N_dL product; this has been done for two different approximations of the stationary electric field in the sample.
Moreover, the noise figures for a reflection type amplifier utilizing the GaAs diode as negative resistance have been calculated in the two aforementioned approximations.
As main results, it is shown that the noise increases very rapidly when the bias field approaches the negative resistance region. The noise figure is a decreasing function of the N_dL product; its limiting value for GaAs being 9.6 db.

Résumé
Les caractéristiques du bruit dans des diodes à l'AsGa, polarisées à fort champ, ont été déterminées. Nous avons considéré que ce bruit provenait des fluctuations des vitesses, ce qui correspond au terme de Nyquist à l'équilibre thermique. Nous avons calculé le spectre de bruit en tension aux basses fréquences, en fonction du champ de polarisation et du produit N_dL; ceci a été fait pour deux approximations différentes du champ électrique stationnaire dans le composant.
D'autre part, le facteur de bruit d'un amplificateur à réflexion dont la résistance négative est constituée d'une diode AsGa a été calculé pour les deux approximations ci-dessus.
Nous avons montré, comme principaux résultats, que le bruit augmente très rapidement lorsque le champ de polarisation approche de la région à résistance négative. Le facteur de bruit est une fonction décroissante du produit N_dL; sa valeur limite est 9.6 db pour l'AsGa.

Introduction.
The noise properties of a non linear device cannot be obtained from the impendence measurement.
If we limit ourselves to consider only the noise due to velocity fluctuations, that, at thermal equilibrium, gives rise to the well known thermal noise, under nonequilibrium conditions the Nyquist formula cannot be applied.
We have calculated the voltage noise power spectrum of a diode made with a bulk differential mobility material, as the GaAs. These devices are now being utilized for amplification at microwave frequencies [1] in amplifiers both of reflection or transmission type.

We have calculated, for a reflection type amplifier, the noise figure in two different approximations. In a first case we have considered an uniform static electric field distribution in all the sample; in a second, more realistic case, we have taken into account the static electric field distribution arising from an ohmic cathode contact.

Noise power spectrum.
The noise properties of an active semiconductor device are considered in a paper of Shockley, Copeland and James [2].
The voltage noise power spectrum $S_v (\omega)$ can be
calculated, by limiting ourselfs at an unidimensional problem, from the expression:

$$\frac{S_v(\omega)}{\Delta f} = \int_0^1 A 4q^2 D(E) n(x) \left| \frac{\partial Z_L}{\partial x} \right|^2 \, dx$$  \hspace{1cm} (1)

where:

- $A$ and $L$ are the cross section and the length of the device;
- $q$ is the electronic charge;
- $D(E(x), \omega)$ is defined by:

$$D = \int_0^\infty \left. \mu_x(0) \right| \cos \omega \tau \, d\tau$$

$D_I$, therefore, is the velocity power spectrum that is the Fourier transform of velocity autocorrelation function;

- $n(x)$ is the stationary electron concentration.

By referring to Fig. 1, $Z_L$ can be defined as $\Delta V / \Delta I$, that is the transfer impedance between a current $\Delta I$ injected at $x$ and the terminal voltage $\Delta V$.

Expression 1) can be considered the generalization of the Nyquist formula $S_v / \Delta f = 4 K T_a R$ for an active device [3].

**Low frequency noise power spectrum.**

For the calculation of the noise power spectrum through expression 1) it is necessary to know the solution of the stationary problem. That is, we must know the stationary electric field distribution $E_n(x)$ for different values of the bias current. By utilizing the well known Poisson's and electron continuity equations and knowing the drift velocity versus electric field relation for the considered material, we can obtain the static electric field distribution; from this, knowing also the diffusion versus electric field relation, the $D(x)$, $n(x)$ functions can be calculated. The transfer impedance $Z_{L,\omega}$ can be calculated from the small signal approximation of Poisson's and electron continuity equations; in the last equation we must consider a generation term of the type $\varepsilon (x - x_0) \varepsilon (t)$.

In the low frequency approximation, that is for frequencies much less than those corresponding to transit time, and for ohmic cathode contact ($E_n(0) = 0$) we obtain:

$$\frac{S_v}{\Delta f} = \frac{4 \varepsilon J}{q L^2 N_p} \int_0^L D(E) \cdot (E_x(L) - E_x)^2 \left( \frac{\varepsilon E}{E_n} + \frac{1}{q J N_p} \right) \, dE$$  \hspace{1cm} (2)

Expression (2) gives the voltage power spectrum at low frequency as a function of the material properties (like diffusion coefficient $D(E)$, drift velocity $v(E)$, uniform doping density $N_p$, dielectric constant $\varepsilon$), geometrical factors (sample cross section A and length L) and bias current J.

It is interesting to observe that the stationary solution $E_n(x)$ is needed to obtain only the electric field values at the anode $E_n(L)$ as a function of the bias current J.

We have also considered the noise for the stationary electric field constant through the sample. In this case, that is for $E_n(x) = E_n$, we get (with $\mu = d\varepsilon / dE$ the differential mobility of the material):

$$\frac{S_v}{\Delta f} = \frac{4 D L}{A \mu^2 N_p} \left[ 1 - \frac{2 \varepsilon v}{q \mu N_p L} \left( 1 - e^{-\frac{q \mu nL}{\varepsilon \tau}} \right) \right] +$$

$$+ \frac{\varepsilon v}{2 q \mu N_p L} \left( 1 - e^{-\frac{q \mu nL}{\varepsilon \tau}} \right)$$  \hspace{1cm} (3)

In the computer calculation of (2) and (3) we have considered for the $\varepsilon (E)$ and $D(E)$ the experimental relations given by Ruch and Kino for the GaAs [4].

In fig. 2 it is shown $S_{is} / S_{lb}$ versus the average electric field $E_n = V / L$ in the sample for both the considered approximations and for four different values of the $N_p L$ parameter. We have normalized the noise voltage $S_{is}$, with respect to $S_{lb}$, the thermal noise of the device for low electric field, that is in the region of ohmic behaviour:

$$S_{lb} = 4 K T_a R \Delta f = 4 K T_a L / (q N_p \mu_a A)$$

where $\mu_a$ is the electronic mobility at low electric field.

From fig. 2 we can observe that, in both considered approximations, the noise shows a strong increment when the average electric field approaches the GaAs peak electric field (3 100 V/cm). Moreover, in the constant electric field approximation, all the curves show a sharp maximum at the same electric field ($\sim 3800 V/cm$). This electric field corresponds to the maximum of the negative differential mobility. If we compare the curves corresponding to the same $N_p L$ product, we can observe that for large $N_p L$, the noise, in the constant electric field approximation, is larger than that corresponding to non uniform
field, mainly for average electric field around the GaAs peak electric field (3100 V/cm). This difference decreases with the $N_{dL}$ product. For large bias field the noise is lower for the uniform field approximation. To completely characterize the device noise behaviour at low frequencies, it is necessary to calculate also the device small signal impedance $Z_{le}$. For brevity sake we do not report the behaviour of the impedance with the bias field.

**Reflection type amplifier noise figure.**

For a reflection type amplifier utilizing the GaAs diode as negative resistance element, the noise figure $F$ or the noise measure for high power gain is given by [5]:

$$ F - 1 = \frac{S_0(\omega)}{4KT_0 |\text{Re} Z_{le}(\omega)|} $$

(4)

We have considered the realistic case in which the stationary electric field distribution is nonuniform.

With the boundary condition $E_o(0) = 0$, that physically corresponds to an ohmic cathode contact, we find a nonuniform stationary electric field distribution for all the bias current.

We obtain for $F$ the expression:

$$ F - 1 = \frac{q^2\varepsilon}{KT_0} \left[ \frac{J - qN_d v(y)}{v(y)} \right] \left[ \frac{J - qN_d v(z)}{v(z)} \right] \exp \left[ \frac{1}{2} \int_0^L \frac{dx/v(x)}{\int_0^L dx/v(x)} \right] $$

(5)

$$ F - 1 = \frac{q^2\varepsilon}{KT_0} \left[ \frac{J - qN_d v(y)}{v(y)} \right] \left( \frac{\int_0^y \cos \left( \frac{\omega T}{2} \int_0^x dx/v(x) \right) \int_0^y dx/v(x) \right) \left( \frac{J - qN_d v(z)}{v(z)} \right) ~ dy $$
where the transit time $T$ is given by $\int_0^t \frac{dx}{v(x)}$

It can be observed from (5) that $F - 1$ is a function of the $N_dL$ product; it is sufficient in fact to consider the new variable $y = N_dL$ and to observe that, being $N_d$ constant, the stationary solution can be written as a function of $N_dL$.

In the nonuniform electric field approximation we have an optimum frequency for which $F - 1$ has a minimum in the negative resistance range.

**Fig. 3.** Optimum value of $F-1$ in the ohmic cathode contact approximation, versus average bias field for four values of $N_dL$.

**Fig. 4.** Optimum value of $F-1$ versus $N_dL$ product for the two cathode contact approximations.
In fig. 3 it is shown the resulting optimum noise figure \( F - 1 \) versus bias field for the already considered \( N_{nL} \) values.

We can observe that the \( F - 1 \) has an optimum with the bias electric field; moreover its value decreases with \( N_{nL} \).

The same calculations for the noise figure can be done for the uniform field approximation. Also in this case we can optimize the noise figure, for a definite \( N_{nL} \) product, also as a function of the average electric field. In the limit of \( N_{nL} \to 0 \) the noise figure is given by:

\[
(F - 1)_{\text{min}} = \frac{q}{kT_0} \frac{D(E_0)}{|\mu(E_0)|}
\]

where \( \mu(E_0) \) is the negative differential mobility.

In fig. 4 we have compared the optimum noise figure in the two considered approximations of uniform and nonuniform electric field as a function of the \( N_{nL} \) product.

As we have before observed the noise figure always decrease with \( N_{nL} \); the improvement however is very small below \( N_{nL} = 5 \times 10^{10} \text{cm}^{-2} \). For the same \( N_{nL} \) value the cathode boundary condition that gives rise to the lower noise figure is that corresponding to constant electric field. The difference is however small being less than 2 db.

Conclusions.

The noise properties of GaAs diodes, biased with high electric fields, have been calculated.

The calculated low frequency, voltage noise spectrum (and eventually device impedance) may permit to compare easily with experiments, the noise behaviour of diodes, done with materials like GaAs, for non thermal equilibrium conditions.

The noise figure of negative resistance amplifier utilizing GaAs as active element has been calculated; the best results can be obtained with low doped diodes \( (N_{nL} \leq 5 \times 10^{10} \text{cm}^{-2}) \). The noise figure depends on the ratio \( \frac{D(E_0)}{|\mu(E_0)|} \) for GaAs 10 db seems the best result that can be obtained.

The calculations have been done considering two extreme conditions for the cathode contact behaviour; the first leading to an uniform stationary electric field in the sample; the second to an always increasing field following zero electric field at the cathode.

The optimum noise figure is always better for the first uniform case; the difference, however, is relatively small, being less than 2 db. Therefore, we can conclude that the optimum noise figure is not a sensitive function of the diode contacts and depends heavily on the material properties mainly on the ratio \( D \mu \) between diffusion constant and negative differential mobility.

Sources


Discussion

J.P. Nougier. — Je désirerais obtenir, en particulier, des éclaircissements sur les deux points suivants:

a) La densité de porteurs \( n(x) \) de la formule (1) dépend manifestement de \( E \). Comment est-elle reliée à la concentration de donneurs \( N_d \) de la formule (2)?

b) Dans l'hypothèse d'un champ électrique uniforme, les deux bornes de l'intégrale de l'équation (2) sont égales et, par conséquent, cette intégrale est nulle. Comment, dans ces conditions, peut-on déduire la formule (3)?

V. Svelto. — Pour la première question : ce qui est constant, c'est la densité des dopes; ce n'est pas la densité des électrons \( n \) qui est fonction des coordonnées.

Pour la deuxième question : l'expression (2) de ma communication sert uniquement au calcul du spectre de bruit lorsque le champ n'est pas uniforme, dans le cas contraire (cas d'un champ égal à la cathode et à l'anode), il faut utiliser une autre expression que je n'ai pas fait figurer dans le texte.

A. De Caquerry. — Perdez-vous que les courbes de minimum du facteur de bruit que vous présentez, puissent avoir un intérêt pratique alors que dans les structures sandwich, pour les dopages et les dimensions usuelles des diodes à l'AsGa, il est vraisemblable qu'une part importante de la diode se trouve en régime de charge d'espace et que, par conséquent, vos hypothèses de champ uniforme ou de champ nul, doivent être très loin de la réalité.

V. Svelto. — We have not considered zero electric field and we have also considered not uniform electric field; the two cases, uniform and non uniform electric field are considered as extreme cases.