Adaptive Noise Shaping ADC Based on LMS Algorithm

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Abstract

A novel technique for achieving noise shaping in oversampled converters is proposed. It uses the adaptive algorithm to minimize the mean square quantization error. The circuit implementation looks like a sigma-delta modulator and requires limited additional circuits for the necessary multipliers. Simulation results, confirmed by experimental measurements, show that more than 15dB in the SNR are gained with respect to an equivalent sigma-delta counterpart in the low signal level range.

1. Introduction

Oversampled noise shaping converters are widely used to trade speed with resolution. Various architectures are proposed and all of them achieve a noise shaping function like \((1 - z^{-1})^N\) where \(N\) is the order of the modulator. The techniques proposed in this paper lead to an architecture that is similar to a sigma delta structure but it is based on different basic idea. We move from the least mean square (LMS) algorithm and we have adapted the resulting signal processing diagram to achieve a circuit that can be implemented with conventional analog basic blocks. The resulting architecture has been extensively simulated at the behavioral level. A CMOS 1,2\(\mu\) double-poly integrated version has been experimentally tested. Results confirmed the validity of the proposed techniques, making it possible to improve the SNR significantly with respect to a conventional sigma-delta architecture [1] in the low signal level range.

2. Adaptive Algorithm

Adaptive signal processing has been contributed in a few decades. A number of applications have been developed so far. Some instances are echo cancellation [2], noise cancellation, signal prediction, system identification, and control [3]. The effect of adaptive algorithms on the performance of the system is analyzed by three components: definition of the objective function \((J)\), definition of the minimization algorithm, and definition of the error signal. Minimization algorithm is the main subject of optimization theory, and essentially the choice of algorithm affects the speed of convergence and computational complexity of the adaptive process. There are several ways to define an objective function that is generally a function of the input \(x(n)\), reference \(d(n)\), output \(y(n)\), and that satisfies optimality and non-negativity.

\[
\text{Optimality : } J(x(n),d(n),e(n)) = 0 \quad (1)
\]

\[
\text{Nonnegativity : } J(x(n),d(n),e(n)) \geq 0 \quad (2)
\]

Adaptive process attempts to minimize the function \(J\), in such a way that \(y(n)\) approximates \(d(n)\) as a consequence, the weight signal, \(w(n)\), converges to \(w_0\), an optimum value of coefficient. The choice of the error signal is quite related to the objective function and is generally taken as difference between reference signal \(d(n)\) and output signal \(y(n)\).

Figure 1 represents the general system involving adaptive algorithm. If we consider that the system is a quantizer with the input signal which is oversampled, then input signal will not vary much during certain amount of time that is \(f_s/2f_b\), where \(f_s\) is sampling frequency and \(f_b\) is signal bandwidth. The coefficient, that the weight signal generates, approaches the optimum value which makes the quantization error to be minimum at every time step. Therefore the total error must be smaller than quantizer with a fixed coefficient, and intuitively increasing sampling frequency affects on how much coefficient approach the optimum value in such a way that total quantization noise decreases in \(f_s/2f_b\). The next subsection describes the LMS algorithm which was applied to this paper. This algorithm makes a realization simple.
2.1. LMS Algorithm

In designing an adaptive system, important procedure is to find the vector \( w(n) \) at time \( n \) that minimizes the objective function. The steepest descent method searches for the optimum coefficient in the iterative procedure. The vector, \( w(n) \) is designed to bring \( w(n) \) closer to the desired solution. This correction involves taking the step size \( m \) that affects the rate at which the weight vector moves down to the minimum error and must be a positive number. The step size decides the speed to approach the minimum error, but there are upper limitation. Using the large value more than this limitation will cause that system to be unstable and unbounded. The updating equation is

\[
w_{n+1} = w_n - \mu \nabla_w J(n).
\]

In general, the evaluation of gradient vector involves finding the expectation \( E[e(n)x^*(n)] \). The general form of the updating equation is

\[
w_{n+1} = w_n - \mu E[e(n)x^*(n)].
\]

The special case occurs if we use one point sample mean. That is, by setting \( E[e(n)x^*(n)] \) equal to \( e(n)x(n)^* \), updating equation becomes more simple.

\[
w_{n+1} = w_n - \mu e(n)x^*(n),
\]

and it is known as the LMS algorithm [4]. The LMS algorithm has been applied to analog-to-digital converters. This approach is discussed in next section.

3. Proposal

The quantizer generates the quantization error which is not only inherent but also unavoidable. To minimize the mean square error generated in the quantizer, we applied LMS algorithm to the quantizer. The important procedure to realize the LMS algorithm is to build the block in Figure 1 which generates weight signal, \( w(n) \).

This weight signal is the input signal of the multiplier shown in figure 2(a). Figure 2(b) includes the model of the quantizer, where the quantizer is modeled in an adder which has two inputs, signal and quantization error. Note that the difference between \( x[n] \) and \( y[n] \) is the quantization error. We defined the objective function, which is basically the square of the error at time \( n \)

\[
J = e^2[n] = (x[n] - y[n]).
\]

From Figure 2 (b), output of the quantizer is

\[
y[n] = w[n]x[n] + N.
\]

To get the expression of the weight signal, we first substitute the equation 7 into equation 6, and take the derivative of the objective function with respect to \( w[n] \).

\[
\frac{\partial J[n]}{\partial w[n]} = -2e[n]x[n],
\]

We can find the minimum value of the objective function with respect to \( w[n] \) by setting \( \nabla_w J[n] = 0 \), which yields the solution of the Wiener-Hoff equation. It can be quite complex to realize the solution of Wiener-Hoff equation in some applications such as high speed systems, therefore we adopt the steepest descent method mentioned in previous section to simplify the realization of the weight signal.

We assume that noise \( N \) and weight signal are independent. This assumption can be a improper assumption in certain situations, but if we have a busy signal as an input, then it is applicable. Therefore equation 8 is simplified to

\[
\frac{\partial J[n]}{\partial w[n]} = -2e[n]x[n].
\]

Actually, the weight signal is a function of time. Therefore, weight signal is derived by applying the Chain rule in the equation 9, which yields

\[
\frac{\partial J[n]}{\partial n} = -2e[n]x[n] \frac{\partial w[n]}{\partial n}.
\]

Although there are several methods to solve equation 10, one of the simplest solutions is found by insight. The derivative of the objective function must be decreasing in time when it has a negative sign. In order to keep objective function negative, we set

\[
\frac{\partial w[n]}{\partial n} = e[n]x[n].
\]

In equation 11, taking integration on both sides, gives the weight signal as in equation 12.

\[
w[n] = \int e[n]x[n]dn
\]

With equation 12, we obtain possible structures for adaptive ADC. Figure 3 shows its discrete time implementation. The adaptive ADC depicted in Figure 3 consists of two multipliers, an integrator and a quantizer. However, having two multipliers in the circuit can be problematic.
We can reduce that number to 1 observing that our approach is based on the sign of the derivative function. In other word, by making negative the sign of the objective function we can achieve a decrease in quantization error at time $n$. Therefore the terms in the right side of the equation 10 that do not affect the sign can be removed. This kind of approach was reported in several articles [5] [6]. In fact, it is difficult to set error signal to have positive sign in this case due to the larger boundary of output of the DAC than one of input signal. But we can easily set the sign of input signal positive by adding an offset. We can then remove the term $x(n)$ in the right side of the equation 10. In order to cancel the effect of offset in the input signal, all of comparator’s thresholds in the quantizer should be shifted by the value of added offset in input signal. Equation 10 is modified to

$$\frac{\partial J[n]}{\partial n} = -2e[n] \frac{\partial w[n]}{\partial n}.$$  \hspace{1cm} (13)

From equation 13, a new representation for weight signal is obtained.

$$w[n] = \int e[n] dn$$  \hspace{1cm} (14)

For the realization of equation 12, we need a multiplier and integrator, but only an integrator is necessary for the equation 14. This simplification is very helpful in higher order modulator implementaton with respect to cost and power consumption.

Using this derived equation, we derive a simplified structure, which is depicted in figure 4. This basic structure has been used for the simulations and implementation. In the next section of this paper, a second order adaptive ADC and the sigma-delta counterpart is discussed.

### 4. Second order adaptive ADC and simulation

The second order discrete time adaptive ADC is shown in Figure 5.

It consists of the cascade of two adaptive integrators connected together similarly to a second order sigma-delta ADC. The gain blocks $m$ are essential to optimize the performances of the circuit. The LMS algorithm suggests to use large values of $m$ but, by contrast, stability issues recommended an upper limit to $m$. We found optimum value of $m$ with extensive simulations. Figure 6 shows that we have a maximum at $m = 1.1$ with a reasonable safety region.

Figure 7 shows the PSD of the proposed circuit and the one of a second order sigma-delta ADC in the normalized frequency range $10^{-1} \sim 10^{-1}$. The input sig-
Figure 8. Simulated SNDR vs. input level

Figure 9. Die Photo

Figure 10. Measured Power spectral density

tions until 40kHz. However, below 40kHz the analog noise coming from the experimental set-up degrades the spectrum in the low frequency range. It was therefore necessary to perform the FFT over the digital version of the output bit stream. Results are marked on Figure 8. The modulator was designed to operate with 3.3V supply.

Figure 9 Shows a microphotograph of the chip. The modulator was designed to operate from 2.5V supply.

6. Conclusion

This paper presented a novel approach to achieve quantization noise shaping. We have shown that the adaptive signal processing algorithm properly applied to the quantization noise, leads to a circuit architecture that resembles a sigma-delta modulator. However, the adaptive solution works better than a sigma-delta for low level signals. Simulations and experimental measurement show that the benefit achieved is 15dB or better.

7. References


