On the Stability of Transferred Electron Amplifiers (*)

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We have considered the small signal impedance and stability of bulk negative mobility diodes having a series resistance $r$ and a parallel capacity $C_p$.

It is shown that the presence of a parallel capacity can lead to instability also with an infinite series resistance. Practical considerations are worked out.

Different authors [1] have calculated the small signal impedance of a diode characterized by differential bulk negative mobility. Under approximate conditions, that is, ideal cathode contact and spatially constant bias electric field, this impedance has the value:

\[
Z(p) = \frac{1}{r V_n A} \left( \frac{e^{-p} - p^2 - 1}{p^2} \right)
\]

We have calculated the effects on the stability of a composite network (fig. 2) obtained by adding to the diode a series resistance $R$ and a parallel capacity $C$ (which takes, for instance, into account the parasitic capacity of the diode encapsulation).

![Diagram](image)

**Fig. 2.** Network considered with $Z$, small signal impedance of the bulk negative mobility amplifier.

The resistance alone by changing the bias supply from a voltage to a current source will produce a stabilizing effect.

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The opposite behaviour can be anticipated for the capacity because of its property of maintaining constant voltage across the diode.

The impedance \( Z_1(p) \) is given by:

\[
Z_1(p) = \frac{I_p}{sV_A} \left( e^{-s\tau} + R + sC_1(p - \tau/s) - p^2 \right)
\]

where we have introduced normalized resistance \( R = \frac{R}{sV_A} \) and capacity \( C_1 = C_1(sV_A) \).

The impedance \( Z_1(p) \) now has new poles when \( C_1 \to 0 \). So, it can be short and/or open circuit unstable if at least a zero and/or a pole has in the \( s \)-plane the position of a real axis. As far as the short circuit stability is concerned, we must consider the zeros of \( Z_1(p) \).

For \( C_1 = 0 \) we have calculated for different values of \( r \) the positions of the zeros in the \( p \)-plane.

In fig. 1 we show the root-locus of the numerator of (2), by limiting ourselves to the first seven singularities, calculated with the zero positions for five different values of \( r \) (\( r = 0, 0.5, 0.7, 4, 8 \)). Naturally, for \( r = 0 \) we obtain as starting points of the root-locus, the zeros of the \( Z(p) \), given by (1).

By increasing \( r \) the zeros shift to the left thereby confirming the increased short circuit stability. It is interesting, also, to note that the first zero is always the last that determines the stability of the system, through its highest real part. In the more general case of \( r \) and \( C_1 \) different from zero, we have calculated from the numerator of (2), the conditions for the first zero to cross the imaginary axis in the \( s \)-plane. We have represented this result in fig. 3, showing \( T/\tau \) as a function of \( r \), for different values of \( C_1 \).

![Fig. 3. -- T/\tau versus normalized series resistance r, for different values of the normalized parallel capacity C1, showing the regions of short circuit stability (S) and instability (I).](image)

For each \( C_1 \) value the stable region is the upper one.

Note that the \( T/\tau \) versus resistance \( r \) plane is divided into two regions: short circuit stable and unstable for each value of \( C_1 \).

It is interesting to observe that a small value of the parallel capacity \( C_1 \) indicates the instability region. Moreover, for a finite \( C_1 \) value, it is impossible, for any value of \( r \), to stabilize a diode with a \( T/\tau \) magnitude lower than some critical value. This is related to the open circuit stability of the system, that is to the poles of expression (2). In fig. 4 we show the relation \( T/\tau \) versus \( C_1 \), representing the condition necessary for a pole of expression (2) to cross the imaginary axis in the \( s \)-plane.

![Fig. 4. -- T/\tau versus C1, showing the open circuit stability of the considered network.](image)

The curve divides the plane into stable and unstable regions. We note that an open circuit unstable diode (due to the presence of a parallel capacity) is always short circuit unstable.

As a conclusion, from fig. 3 and 4, it is possible to calculate for a definite diode, for each \( T/\tau \) value, the values of the maximum parallel capacity and the minimum resistance necessary to obtain a small signal stable circuit. We can estimate the importance of this effect on real diodes by noting that the usual parasitic capacity of microwave devices mounted in modified IN23 packages is about 0.5 pF. For an X-band Gunn diode with an area of 250 \( \mu \)m \( \times \) 250 \( \mu \)m and a thickness of 10 \( \mu \)m the normalized capacity \( C_1 \) would then be about 20. This indicates that it would be almost impossible to stabilize a supercritically doped diode by a pure resistive load.

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**REFERENCES**

