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High Frequency Performance Limitations of Low-Pass Anti-Aliasing Filters

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Abstract. The effects of operational amplifier non-idealities, parasitic capacitances and substrate resistances on the high frequency response of low-pass anti-aliasing filters are considered.

1. INTRODUCTION

A low-pass anti-aliasing filter is always used in sampled-data systems in order to avoid aliasing effects. The cut-off frequency of the filter must be located between the upper edge of the input signal band and one half of the sampling frequency.

Typically, the filter must provide at least 30 dB of attenuation at the sampling frequency, and have a flat response over the input signal band. For this purpose, a second order transfer function with pole-Q = 0.707 is generally chosen (Butterworth low-pass characteristic).

In order to limit the chip area occupation of the anti-aliasing filter, topological schemes with only one active element are often used, since single operational amplifier filters may also display low sensitivity with respect to passive component variations.

Today's trend is to increase the clock frequency of the sampled-data processors, thus obtaining an increase of the processed signal band. Therefore, the cut-off frequency of the anti-aliasing filter must also increase. However, when operating at high frequency, the anti-aliasing filter response is modified by the operational amplifier's non-ideal performances and by parasitic effects; remarkable differences from the ideal case can result.

In this paper, the limitations due to the finite DC gain, the finite bandwidth and the non-zero closed-loop output resistance of the op-amp are analyzed. Moreover, parasitic effects associated with diffused layer resistors are considered.

2. SINGLE OP-AMP ANTI-ALIasing FILTERS

The Sallen & Key (S&K) [1] and Rauch [2] topologies are used commonly to realize a second order anti-aliasing filter with single op-amp.

The Sallen & Key scheme is shown in Fig. 1a. If all the elements in the circuit are ideal, the filter transfer function is:

\[ \frac{V_{out}}{V_{in}} = \frac{1}{1 + s(R_1 + R_2)C_2 + s^2 R_1 R_2 C_1 C_2} \]  

A maximally flat response can be obtained by choosing \( R_1 = R_2 = R \) and \( C_1 = 2C_2 = 2C \), with cut-off frequency \( f_c = 1/(2\pi \sqrt{2RC}) \). An advantage of the S&K filter is that, at low frequencies, no current flows into the resistors; therefore non-linear diffused layers with high sheet resistance can be used, hence obtaining a large area saving without generating low frequency harmonic distortion [3]. On the other hand, a drawback of this scheme is that the dynamic range of the filter is limited by the op-amp common mode input swing, since the op-amp is in the buffer configuration.

The Rauch scheme is shown in Fig. 1b. Its ideal transfer function is:

\[ \frac{V_{out}}{V_{in}} = \frac{1}{1 + sR_1 C_1 + s^2 R_1 R_2 C_1 C_2} \]
From Eq. (2), it can be seen that the low-frequency gain of the filter is given by the ratio \( \frac{R_2}{R_1} \). Setting this gain equal to 1, a Butterworth characteristic can be obtained by choosing \( R_1 = R_2 = 2R \) and \( C_1 = 4C_3 = 4C \), with \( f_e = 1/(4\pi \sqrt{2RC}) \). In this scheme, the non-inverting input of the op-amp is directly connected to ground; thus, owing to the presence of the virtual ground, the operational amplifier is not affected by common mode input signals. On the other hand, low-frequency distortion, due to the voltage drop across them.

3. Sallen & Key Filter with Real OP-AMP

The operational amplifier generally used in CMOS monolithic implementations of S&K filters is a transconductor amplifier, since the gain is typically loaded by a sample-and-hold circuit acting as a capacitive load. At high frequency the op-amp does not operate anymore as an ideal device, and its finite gain, bandwidth and output resistance must be taken into account.

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_1}{R_1 + s\left(\frac{R_1 R_2}{R_1 + R_2} + s^2 R_1 R_2 C_1 C_2\right)}
\]

\[
\gamma = \frac{2R_e^2 C^2}{A_0} + \frac{4R_e R_0 C}{C_0} + \frac{C_0}{A_0} + \frac{2R_e^2 C^2}{A_0}
\]

\[
\delta = \frac{2R_e^2 C^2}{\omega_f}
\]

The frequency response has two zeros and three poles. Putting \( 2G_m R = K \), if the transconductance gain, \( G_m \), of the op-amp is such that \( K > 1 \), the zeros are complex. Their real part is determined by fixing the filter design parameters, while their imaginary part depends also on the \( G_m \) value. Under the condition \( K > 1 \), that usually occurs in practical cases, the zero-Q is high, hence the frequency response displays a notch effect approximately located at the frequency \( G_m / (2\pi C \sqrt{K}) \).

One of the poles is always real and is located around the unity-gain frequency of the operational amplifier. Because of op-amp bandwidth limitations, the two remaining complex poles are shifted to a lower frequency with respect to the design position. The shift of the complex poles depends on the factor \( K \) and on the op-amp unity-gain frequency, \( f_f = G_m / (2\pi C_0) \), for a given design value of the cut-off frequency, \( f_c = 1/(2\pi \sqrt{2RC}) \).

A suitable equivalent circuit of the operational amplifier is shown in Fig. 2, with \( G_m \), \( R_0 \), and \( C_0 \) given by the following relations:

\[
G_m = \frac{A_0}{R_{\text{cm}}}, \quad R_0 = R_{\text{cm}}, \quad C_0 = \frac{A_0}{R_{\text{cm}} \omega_f}.
\]

where \( A_0 \), \( R_{\text{cm}} \), and \( \omega_f \) are the low-frequency gain, the output resistance and the gain-bandwidth of the op-amp, respectively.

By using this equivalent scheme, the S&K filter transfer function becomes:

\[
F(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\alpha + s\beta + s^2\gamma + s^3\delta}
\]

\[
\alpha = 1 + \frac{1}{A_0}
\]

\[
\beta = 2RC + \frac{R_0 C_0 + 2R_0 C + 4RC}{A_0}
\]

\[
\gamma = \frac{2R_e^2 C^2}{A_0} + \frac{4R_e R_0 C}{C_0} + \frac{C_0}{A_0} + \frac{2R_e^2 C^2}{A_0}
\]

\[
\delta = \frac{2R_e^2 C^2}{\omega_f}
\]

Fig. 3. a) Module of the S&K poles, versus \( f_c \).

b) Q of the S&K poles, versus \( f_c \).

Fig. 2. Op-amp equivalent circuit.
The module and the Q of the complex pole pair as a function of the normalized unity-gain-frequency, \( f_p \), are plotted in Fig. 3a and 3b respectively for three different values of the factor \( K \) (\( K = 100, 10 \), and 3). The design value of the poles' module is 200 kHz. A significant deviation in this module in and the value of the Q from the design position should be noted, as both factor \( K \) and the \( f_p \) ratio decrease. Three S&K filter frequency responses, as simulated numerically with SPICE [4], are shown in Fig. 4. Curve A, B and C refer to \( f_p / f_0 = 100, 20 \) and 4 respectively; while \( K \) is set to 40 for each curve. The design value of \( f_p \) is 200 kHz. This figure shows the dependence of the high-frequency attenuation and of the filter cut-off frequency on the \( f_p / f_0 \) ratio.

4. RAUCH FILTER WITH REAL OP-AMP

The high frequency performances of the Rauch filter (Fig. 1b) can be derived by using again the equivalent circuit of the operational amplifier shown in Fig. 2. The transfer function becomes:

\[ F(s) = \frac{V_{out}}{V_{in}} = \frac{-1 + s}{\alpha' + s\beta' + s^2\gamma' + s^3\delta'} \]

\[ \alpha' = 1 + \frac{4}{A_0 - 1} \]

\[ \beta' = 4R_C + \frac{2R_0C_0 + R_0C + 13RC}{A_0 - 1} \]

\[ \gamma' = 8R^2C^2 + 6R_0R_CC_0 + 2R_0C^2 + 18R^2C^2 \]

\[ \delta' = \frac{8R_0^2C_0^2}{A_0 - 1} \]

A comparison between the simulated and the measured S&K filter frequency responses is shown in Fig. 5. The measurement was carried out on a S&K filter built with discrete components and a test pattern two-stage transconductance operational amplifier, which was integrated in a 3.5 \( \mu \)m CMOS process. The op-amp electrical parameters were \( G_m = 1 \text{mA/V}, R_0 = 1 \text{M} \Omega \), and \( C_0 = 100 \text{pF} \).

The used resistance and capacitance values were \( R = 18 \text{k}\Omega \) and \( C = 22 \text{pF} \) respectively, giving \( f_p = 284 \text{kHz} \).

The substantial similarity displayed by the two curves demonstrates the validity of the model used.

Fig. 6 - a) Module of the Rauch poles, versus \( f_p / f_0 \).

b) Q of the Rauch poles, versus \( f_p / f_0 \).

The frequency response has one zero and three poles. The zero's position is \( f_p = |A_0 - 1|/2\pi R_0 + R C \); it is far away from the design poles' position so its
effect is unimportant. If \( f_1 > f_p \), that is \( G_{op} / (2\pi C_0 \approx \frac{1}{(4\pi \sqrt{2R C})}) \), one of the poles is real and is located around the op-amp unity gain frequency, \( f_p \); while the two remaining poles are generally complex. When a real op-amp is considered, the complex poles move towards the real axis of the complex plane, as \( f_p \) approaches \( f_1 \).

The module and the \( Q \) of the complex pole pair as a function of the normalized unity-gain frequency, \( f_1 / f_p \), are plotted in Fig. 6a and 6b respectively, for three different values of the factor \( K \) (\( K = 100, 10 \) and 3). The design value of the poles' module is 200 kHz. The curves show that, for low \( f_1 / f_p \) ratio less than 20; a small change in \( f_1 / f_p \) can result in a large variation of the module and the \( Q \) of the pole pair. Moreover, for very low values of the \( f_1 / f_p \) ratio less than 1.5 approximately, the poles become real.

Fig. 7 shows the Rauch filter frequency response for the same values of the parameters \( K \) and \( f_1 / f_p \) as were used for the plots in Fig. 4. As foreseen, the reduction of \( f_1 / f_p \) determines a lowering of the filter cut-off frequency.

The experimental measurement done on the S&K filter was also carried out on the Rauch filter. The same op-amp was used, while the resistance and capacitance values were \( R = 50 \Omega \) and \( C = 6.8 \mu F \) respectively (\( f_p = 165.5 \) kHz).

5. EFFECT OF THE INTEGRATED RESISTORS

In integrated circuits, resistances are made with strips of polysilicon or diffused layers. A lumped element model for these integrated resistors is only valid as a first hand approximation, since they are capacitively-coupled with the substrate. A better model is given by a distributed element \( R \) \( C \) network [5]. Fig. 9a.

![Fig. 9](image)

(a) Distributed model for a diffused resistor
(b) Equivalent II circuit of a diffused resistor

Its equivalent II network is shown in Fig. 9b. The series and shunt admittances \( Y \) and \( Y_p \) are:

\[
Y = \frac{\sqrt{s} C / R}{\sinh \sqrt{s} RC}
\]

\[
Y_p = Y \cosh \sqrt{s} RC - 1
\]

A more accurate analysis of the S&K and Rauch filter performances can be accomplished by using the resistor model described in Fig. 9, together with the op-amp model shown in Fig. 2. Moreover the substrate terminal must be assumed connected to a fixed voltage through a non-zero resistance, because the substrate resistivity is not negligible. With these models, the complexity of the networks to be analyzed is such that only numerical evaluations are possible.

![Fig. 10](image)

Simulated frequency responses of a S&K filter, obtained by using the distributed resistor model.

A computer program for the analysis of the anti-aliasing filters was written and the frequency responses for practical cases were determined. Typical diffused layer parasitic capacitance of 0.15\( \mu F / \mu m^2 \), sheet resistance of 3\( \Omega / \mu m \), and resistor strip width of 20\( \mu m \) were considered. Fig. 10 and 11 show the frequency
responses of the S&K and of the Rauch filter respectively. The curves were obtained by using the previous models, assuming $J_t/J_p = 20$ and $K = 40$, for three different values of the simulated substrate resistances: 1 kΩ (curve A), 5 kΩ (curve B), and 10 kΩ (curve C).

![Fig. 11. Simulated frequency responses of a Rauch filter, obtained by using the distributed resistor model.](image)

A comparison of the three frequency responses with the ones in Fig. 4 and 7 shows a worsening of the high frequency attenuation. This is due to the substrate resistance which gives a high frequency coupling between the input and the output terminals. This coupling can be reduced if the substrate resistance is lowered. One way to achieve this is to bias the substrate all around each diffusion strip used to realize the resistances.

![Fig. 12. Measured frequency response of an integrated Sallen & Key filter.](image)

A S&K integrated filter frequency response is shown in Fig. 12. The filter is realized in a 3.5 μm double poly CMOS process. The resistors were 20 μm wide $p$-guard diffusion layers with a resistivity of 3.5 kΩ/μm. The op-amp was a transconductance folded cascode amplifier. As we can see, this frequency response shows a notch effect due to the complex zeros, and a high frequency flattening due to the parasitic capacitances and substrate resistances.

6. CONCLUSIONS

The performances of S&K and Rauch filters with a transconductance operational amplifier and diffused layer resistances were analyzed, by using simple equivalent models. Op-amp performance limitations and distributed capacitance effects were considered. Simulations show that, in high frequency range, anti-aliasing filter frequency responses can be quite different from the expected ones, if real element limitations are considered.

Comparison between simulated and measured results has also been reported.

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REFERENCES


