A Direct-Digital Synthesizer with Improved Spectral Performance
Paul O’Leary and Franco Maloberti

Abstract—This paper presents a modified direct-digital synthesizer which uses noise shaping to reduce the effects of phase-accumulator truncation on the output spectrum. The discrete spectral disturbances associated with this truncation error are strongly reduced with the proposed method, making the synthesizer suitable for high performance signal processing. The proposed architecture uses first-order noise shaping. Higher order noise shaping can also be used if required.

I. INTRODUCTION

In recent years direct-digital synthesis has been applied in many areas [1]–[8]. The spectral purity at the output is important in most applications, especially where the synthesizer is used in nonlinear processes.

In most applications the accumulator has a large bit width, typically 16–20 b. From the accumulator only a small number of bits are used as address for the Sine ROM, this corresponds to a phase truncation. The error due to this phase increment discretization (phase truncation) and due to the finite ROM data word are usually described in terms of a noise contribution. The power associated with that noise should have a white spectrum. However, with conventional direct-digital synthesizers (DDS), a white noise component is often accompanied by discrete lines. The presence of discrete spectral lines in the output spectrum is undesirable because they can result in intermodulation components. To avoid this undesirable effect, the conventional solutions dimension the sine ROM such that the output noise is dominated by the amplitude quantization and not by the phase truncation. Unfortunately, for high-precision sine waves the required ROM becomes prohibitively large. The use of trigonometrical decomposition [9] offers only a small improvement. The ROM size can also be reduced with the use of complex computation [10] but this is done at the cost of a more complex circuit.

This paper presents a direct-digital synthesizer where a noise shaping technique is applied to the phase truncation to overcome the problems of spurious spurs in the output spectrum. The modified circuit offers considerable improvement in the synthesizer performance when the generated frequency is low with respect to the clock frequency.

II. CONVENTIONAL DIRECT-DIGITAL SYNTHESIZER

Fig. 1 shows the block diagram of a conventional direct digital synthesizer. It consists of an N bit accumulator the output of which is truncated to m bits as the address of a Sine ROM with an L bit wide data bus.

The truncation of the phase word is equivalent to quantization. If it is assumed that quantization can be modelled as a linear additive process then the output of the synthesizer is given by

$$\text{Out}(t) = \sin\left( \frac{2\pi f_{\text{gen}}}{2^m} + \epsilon_p(t) \right) + \epsilon_L(t)$$

(1)

where

- $f_{\text{gen}}$ is the generated frequency.
- $\epsilon_p(t)$ is the error associated with the phase truncation.
- $\epsilon_L(t)$ is the quantization error due to the finite ROM data word.

Using a trigonometrical equality and assuming that the phase error is small relative to the phase, (1) becomes

$$\text{Out}(t) = \sin\left( \frac{2\pi f_{\text{gen}}}{2^m} \right) + \epsilon_p(t) \cos\left( \frac{2\pi f_{\text{gen}}}{2^m} \right) + \epsilon_L(t).$$

(2)

Thus, the phase error is amplitude modulated with the quadrature signal to the desired frequency. Consequently, discrete spectral lines in the phase error will be modulated to discrete spectral pairs in the synthesizer output spectrum.

The relevant contribution to the global error from the phase truncation error is demonstrated with the help of Figs. 2 and 3. Fig. 2 shows the phase truncation error spectrum from a conventional DDS running under the following conditions: $F_{\text{clk}} = 400$ KHz, $f_{\text{gen}} = 1080$ Hz, $N = 16$, $M = 6$, and $L = 14$. Fig. 3 shows the synthesizer output spectrum of the same DDS system. The spectral pairs due to the phase truncation are easily identified.

A thorough analysis of phase truncation error effects on the output spectrum of conventional digital synthesizers has been presented by H. Nicholas et al. [11]. He shows that the worst

Footnotes:

1 All spectra presented in this paper were generated using a 4096 point FFT with Hanning window.
to ensure that the output error is dominated by the ROM data width and not by the address truncation. Unfortunately for high-precision sine waves this requires a very large ROM. (The ROM size increases exponentially in powers of 2 with the address width but only linearly in the number of data bits.)

III. MODIFIED ARCHITECTURE

The phase accumulator is often between 16 and 20 b wide. This corresponds to an almost ideal description of the phase. It has been noted that the truncation of this phase to \( m \) bits as ROM address corresponds to quantization. If the frequency being generated is low with respect to the used clock frequency, then there is an intrinsic oversampling. This oversampling can be used to apply a noise shaping technique [12] to reduce the effects of the phase truncation.

The use of an oversampling technique makes it possible to take into account the individual phase truncations by accumulating the resulting phase error. The total error can now be reduced by suitably correcting the ROM address with the accumulated error. The final result is equivalent to a linear (for first-order noise shaping) or a quadratic (for second-order noise shaping) interpolation between two consecutive ROM addresses.

Fig. 4 shows the architecture of the modified direct digital synthesizer. A first-order noise shaper has been placed between the phase accumulator and the ROM. Using calculation methods similar to those used by J. Candy [12] it is possible to derive the following equation for synthesizer output signal:

\[
\text{Out}(f) = \sin \left( \frac{2\pi f_{gen}}{2^m} \right) + \{ v_p(i) - v_p(i-11) \} + \cos \left( \frac{2\pi f_{gen}}{2^m} \right) + e_L(i). \tag{7}
\]

The additional processing of the phase error is equivalent to a first-order digital high-pass filtering of the phase “noise” before the amplitude modulation through the ROM. The improvement in the signal-to-noise performance for low-frequency signals is evident. More importantly the noise shaper also decorrelates the truncation error eliminating the discrete spectral lines in the output spectrum. Fig. 5 shows the output spectrum from the modified architecture with the same ROM and accumulator as for Figs. 2 and 3 but with the addition of the noise shaping (the synthesizer is running under the same conditions as the conventional synthesizer). It can be observed that a significant improvement in the output spectrum has been achieved. The output spectrum does not display strong discrete spectral disturbances, this makes the synthesizer suitable even for nonlinear applications.

IV. CONCLUSION

A modified direct digital synthesizer has been presented which uses noise shaping to reduce the effects of phase truncation on the output spectrum. It has been demonstrated that this can give a considerable improvement in the performance of the synthesizer, particularly for nonlinear applications. The
Fig. 4. New direct digital synthesis architecture.

Fig. 5. Output spectrum for the new architecture.

The proposed architecture used first-order noise shaping but the use of higher order noise shaping is possible.

REFERENCES


