Noise and gain in a SC integrator with real operational amplifier

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Abstract: An active switched-capacitor integrator has been analyzed taking into account the actual characteristics of the real operational amplifier, as far as noise and frequency response are concerned. Through an equivalent circuit for the SC structure, the response to a sinusoidal input signal is calculated for the inverting and non-inverting configurations; thereafter, the output noise spectrum due to the internal noise sources of the operational amplifier is derived. The correspondence with the analog RC active integrator is shown, pointing out the frequency range of integrating behaviour of the two circuits, and the increased noise due to aliasing effects for the SC version. Experimental measurements of frequency response and output noise spectra for different sampling rates check the theoretical results.

1. INTRODUCTION

Monolithic integration of active filters can be achieved by using switched capacitors and operational amplifiers in MOS technology [1]. Several filter topologies can be used for a monolithic implementation. As a general approach in SC filter design, a particular interest has been devoted to transfer into an appropriate SC version, those classes of analog (active or passive) filters, for which the design procedures have already been well established. In this context, different SC filters have been proposed in which the operational amplifiers are employed as integrators, just as in the analogous RC realization, with the resistors replaced by properly switched capacitors. In particular, we refer to state-variable ladder active filters, or to leapfrog (ladder active) filters, or to three amplifier biquad filters, for which the basic building block is represented by the active operational amplifier integrator [2, 3].

In this work we deal, theoretically and experimentally, with an active SC integrator, in order to characterize its performance as far as frequency response and output noise are concerned, when the operational amplifier is a real one. We compare the results to the RC analogous case under the same assumptions. By assuming a finite dc-gain and a dominant pole in the operational amplifier transfer function, the frequency response of the integrator will be degraded with respect to the ideal case. Moreover, taking into account the operational amplifier noise through a white input noise voltage source, the output noise spectrum will be found to be higher than that in the RC case for aliasing effects, depending on the clock frequency.

For such an analysis, we need to calculate the exact output response of the SC active integrator to a sinusoidal input signal. From that, the frequency response and the noise voltage spectrum are easily derived.

The integrator we have analyzed employs as SC structure that proposed by Temes [4], which achieves a bilinear transformation between analog and sampled cases. For this SC arrangement a model has been proposed [5] useful here to evaluate the general behavior of the SC integrator in a more direct form than that suggested by Xu and Pezzi [6].

Finally, we have measured the actual frequency response and the output noise spectrum due to the internal sources of the operational amplifier. The results agree quite well with the theoretical analysis and are useful in evaluating the overall noise performances of more complex SC filters, at least employing the SC circuit here considered.

2. CIRCUIT ANALYSIS

Before to consider the switched-capacitor circuit under investigation, we examine the behavior of the corresponding continuous circuit, so to have a reference term.

In Fig. 1(a) and 1(b) two simple continuous circuits are shown. The operational amplifier is assumed to be real with transfer function given by:

\[ A = \frac{A_0}{1 + sA_0} \]

Under the usual conditions \( A_0 \gg 1 \), \( RC \gg s/A_0 \), the transfer functions of the two circuits of Fig. 1 are:

\[ A^{(-)} = \frac{v^{(-)}_0}{v^{(-)}_1} = \frac{A_0}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) +} = \frac{v^{(+) +}}{v^{(+) -}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) -} = \frac{v^{(+) -}}{v^{(+) +}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(-)} = \frac{v^{(-)}_0}{v^{(-)}_1} = \frac{A_0}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) +} = \frac{v^{(+) +}}{v^{(+) -}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) -} = \frac{v^{(+) -}}{v^{(+) +}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(-)} = \frac{v^{(-)}_0}{v^{(-)}_1} = \frac{A_0}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) +} = \frac{v^{(+) +}}{v^{(+) -}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

\[ A^{(+) -} = \frac{v^{(+) -}}{v^{(+) +}} = \frac{A_0(1 + j\omega RC)}{(1 + j\omega RC_0)(1 + j\omega A_0)} \]

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Fig. 1 - The active RC circuit for inverting (a) and non-inverting (b) inputs.

If we replace the resistors of Fig. 1 with the switched-capacitor structure suggested by Tenes [4], we obtain the circuits shown in Fig. 2. The circuits of Fig. 1 and Fig. 2 are "almost" corresponding if $R = 1/2C_1$, being $T$ the time between the reversal of the switches.

If the switches are instantaneously reversed at times $nT$, the capacitor $C_1$ is always connected to the circuit and the SC structure can be simulated by the equivalent circuit shown in Fig. 3. It consists of the capacitor $C_1$ (always connected) having in parallel a $\delta$-current generator $i(t)$, simulating its periodical reversing:

$$i(t) = 2C_1 \sum_{n=-\infty}^{\infty} \left[ v_A(t) - v_B(t) \right] \delta(t - nT)$$

The equivalent circuit of Fig. 3 is convenient for a direct analysis in the Fourier domain of the circuits of Fig. 2. This is made in detail in Appendix to obtain the exact response to a general input signal.

For the considered circuits the obtained results are more general than the analogous ones derived in the Z-domain [7]. In fact they can be utilized also with input signals not band limited, as in the case of noise considerations.

For sinusoidal input, that is for:

$$V_s(t) = \cos(\omega_0 t)$$

$$V_s(j\omega) = \sum\{V_s(t)\} = s\pi(\omega + \omega_0)$$

Fig. 2 - The analyzed SC integrating circuit for inverting (a) and non-inverting (b) inputs.

We obtain from (4) the following equations:

$$\frac{1}{C_1} \left[ v_A(t) \right]_{t = \pm T} = -\frac{A}{s\pi} \left[ v_A(t) \right]_{t = \pm T} + \frac{1}{s\pi} \delta(m\omega_0 + ko_0)$$

$$i(t) = 2C_1 \sum_{n=-\infty}^{\infty} \left[ v_A(t) - v_B(t) \right] \delta(t - nT)$$

Fig. 3 - The equivalent circuit of the switched capacitor.
\[ \frac{1}{\tau} y(t) \sim (\Delta u) \bigg|_{\Delta u} = \frac{A}{\lambda + \gamma} \bigg[ y(t + \omega) + \sum_{k=0}^{\infty} A e^{i \omega k} \bigg] \]

To obtain (7) and (8) we remember that, by definition of star operator:

\[ (F(\omega) \delta(\omega - \omega_0))^{*} = F(\omega - \omega_0) \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + k\omega) \]

where \( \omega_0 = 2\pi/T \) is the sampling angular frequency. Moreover, we have introduced:

\[ \gamma = 1 + \frac{C}{C_T} = 1 + \frac{1}{2RC} \]

\[ B = \frac{2(\gamma - 1)}{2(\gamma - 1)} \frac{1}{1 + 2(\gamma - 1) R C} \times K \]

\[ K = \frac{e^{-\gamma} + A}{1 - e^{-\gamma} + A} \]

Equations (7) and (8) show that, in both considered circuits of Fig. 2, the output voltage due to a sinusoidal input at frequency \( \omega_0 \) is composed of a sinusoidal term at the same frequency \( \omega_0 \), and sinusoidal terms at frequencies \( \omega_0 + k\omega \) (\( k = \pm 1, \pm 2, \ldots \)).

This behaviour can be also represented as in Fig. 4 and Fig. 5, where the block diagrams refer to equations (7) and (8), respectively.

**Fig. 4** - Block diagram representation of the response of the circuit of Fig. 2a.

**Fig. 5** - Block diagram representation of the response of the circuit of Fig. 2b.

The analytical expression of \( B \) (equation (10)) does not permit an immediate understanding of its behaviour. This understanding, however, is immediate by examining the plot of the numerical calculation of (10), carried out for values of the parameters \( A_0 \), \( \gamma \), \( \lambda \), \( \tau \), \( RC \), corresponding to real situations. Such a plot is shown in Fig. 6, as calculated for \( A_0 = 10^3 \) and \( RC = 1 \) decade lower than the unity gain angular frequency of the operational amplifier. It results that \( B \) is substantially independent of \( \gamma \) and \( \tau \); it introduces a pole at \( \omega = 1/A_0 \) and has a low frequency gain approximately equal to one, being \( A_0 > > \gamma \):

\[ B(\omega) = \frac{A}{\omega + \frac{1}{RC}} \]

**Fig. 6** - The modulus of \( B(\omega) \) vs the angular frequency. The numerical results are practically coincident for \( C_0C = 0.5; 0.2; 0.1; 0.05 \) and for the requested values of \( \tau \) to maintain the same \( RC \), two decades higher than \( \tau/A_0 \).
Combining the two approximate expressions (11) and (12) of \( A/(A+Y) \) and \( B \), we obtain that the block \( A/(A+Y) \) has approximate response almost equal to (2), that is \( B = A^* \).

To complete the understanding of the blocks of Fig. 4 and Fig. 5, we observe that the prefiltering block consists of a low-pass block with -3 dB a

ular frequency at \( A_0/2\pi \) for the non-inverting configuration, and a block with transfer function approximately 1 in the inverting configuration (for \( Y = 1 \)).

3. FREQUENCY RESPONSE

Equations (7) and (8) allow to determine the inverting and non-inverting frequency response defined as the ratio of the voltage output component at the frequency of the input, to the input sinusoidal voltage. We get:

\[
\begin{align*}
H^* & = \frac{A}{A+Y} \frac{(\gamma-1)}{\gamma} + B \frac{A+1}{A+Y} \\
& = \frac{A_0(1+j\omega T)}{(1+j\omega T_0)(1+j\gamma R\omega_0)}
\end{align*}
\]

(14) \[ H^* = \frac{A}{A+Y} \frac{(\gamma-1)}{\gamma} + B \frac{A}{A+Y} \]

It can be observed that (13) and (14) are derivable directly from the block diagrams of Fig. 4 and Fig. 5 if the ideal samplers are short-circuited.

Comparing (13) with (2), we see that the transfer function of the sampled and analog cases are equal for \( \omega \ll 2/1 \). In fact, (13) has an additional zero for \( \omega = 2/1 \) due to the feed-through path.

The frequency responses given by (2) and (13) are compared in Fig. 7 for \( A_0 = 1000 \), \( RC = 15.6 \mu \text{s} \) and \( A_0/2\pi = 1 \text{ kHz} \), but changing, for the SC case, the values of \( \gamma = 1 = C_1/C = T/2RC \) and therefore for different values of the sampling frequency \( 1/T \).

The dashed line refers to the continuous case, while the full lines are for the SC circuit with \( C_1/C = 0.5; 0.2; 0.1; 0.05 \) starting from the upper curve; accordingly, the sampling frequencies are 62.8 kHz, 157 kHz, 314 kHz, 628 kHz respectively.

In the figure the marked frequencies \( 1/10 \text{ Hz} \) are in correspondence with the zeros of the frequency responses.

In the same Fig. 7 we report the experimental points, obtained employing the full compensated \( \mu \text{A} 741 \) as operational amplifier with a measured \( \text{gain bandwidth} = 1 \text{ MHz} \), and the LF11333 as analog switches. To avoid drift problems, a fixed resistor of 22 \( \Omega \) is put in the feedback path of the operational amplifier; different values of \( \gamma \) are obtained changing \( C \) and accordingly \( T \). We do not show the low-frequency results, being the comparison between theoretical and experimental results not meaningful, due to the presence of the feedback resistor and to the different value of the open loop gain of the used operational amplifier.

We can conclude that for the inverting integrator the useful bandwidths, in the continuous and sampled cases, start always from the same low-frequency value \( 1/(2\pi RC_0) \); the high frequency limits can be different, being always \( A_0/2\pi \) in the continuous case, while in the sampled case is the lower value between \( A_0/2\pi \) and \( 1/10 \text{ Hz} \). So to have a large bandwidth in the sampled case it is useful to have the largest possible sampling frequency \( 1/10 \text{ Hz} = A_0/2\pi \), usually limited from the corresponding ratio \( C_1/C \) technologically allowed.

The non-inverting transfer function (14) is quite coincident with (3) as far as \( \gamma = 1 \). In fact (14) at low frequency equals (6); at high frequency \( (\omega > 1/RC) \) the dominant term is \( A/(A+Y) \) which for \( \gamma = 1 \) equals (3).

The frequency response given by (14) is reported in Fig. 8 for the values of \( C_1/C = 0.5; 0.2; 0.1; 0.05 \) and for sampling frequencies \( 1/10 \text{ kHz} \); 157 kHz; 314 kHz; 628 kHz respectively. The curve for \( C_1/C = 0.5 \) is the upper one, while the others are practically coincident. The frequency response given by (3) (dashed line) is superposed with the lowest curve of Fig. 8. The experimental points reported in the same figure refer to the SC cases and agree quite well with the theoretical results.

4. NOISE

The formulas of the previous sections are useful to evaluate the output noise due to a noise source applied to the input of the circuit or due to the internal noise generator. Here, we will evaluate the response due to the internal noise generator.

A preliminary calculation has shown that the input noise current generator of the operational amplifier gives negligible contribution to the output noise voltage. The equivalent circuit considered for noise calculations is the one shown in Fig. 9, where the voltage noise source is applied...
due to a sinusoidal input signal at frequency \( \omega_0 \), is given by:

\[
V_o(\omega_0) = \frac{A(\omega_0)}{A(\omega_0)^2 + \frac{1}{4\pi^2}} \left( \frac{B(\omega_0)A(\omega_0)}{A(\omega_0)^2 + \gamma} \right) = H^{(+)}(\omega_0) k=0
\]

\[
V_o(\omega_o) = \frac{A(\omega_o)B(\omega_o)}{A(\omega_o)^2 + \gamma + A(\omega_o)^2 + \gamma^2} k=0
\]

Therefore, the output noise spectrum \( S_V(\omega) \) is related to the input spectrum \( S_e(\omega) \) by:

\[
S_V(\omega) = |H^{(+)}(\omega_0)|^2 S_e(\omega_0) + \left| \frac{B(\omega_0)A(\omega_0)}{A(\omega_0)^2 + \gamma} \right|^2 S_e(\omega_0 + \omega_0) + \sum_{k=0}^{\infty} \left| \frac{A(\omega_0 + k\omega_0)}{A(\omega_0 + k\omega_0)^2 + \gamma} \right|^2 S_e(\omega_0 + k\omega_0)
\]

The first term of (15) equals the output noise spectrum of the continuous case, while the second term is the aliasing contribution. This baseband folding originates from input frequencies beyond the Nyquist zone \( \omega \leq \pm \omega_0 \). Letting \( \gamma = 0 \), we get:

\[
S_V(\omega) = \left| H^{(+)}(\omega_0) \right|^2 S_e(\omega_0) \left| \frac{B(\omega_0)A(\omega_0)}{A(\omega_0)^2 + \gamma} \right|^2 S_e(\omega_0 + \omega_0)
\]

where:

\[
N(\omega) = \sum_{k=0}^{\infty} \left| \frac{A(\omega_0 + k\omega_0)}{A(\omega_0 + k\omega_0)^2 + \gamma} \right|^2
\]

In the frequency range \( \omega < 2\pi/T \) of usual utilization of the integrator, \( B/A(\gamma) = H^{(+)} \) and (16) can be approximated by:

\[
S_V(\omega) = \left[ 1 + N(\omega) \right] \left| H^{(+)}(\omega_0) \right|^2 S
\]

For the continuous case (Fig. 1) it is immediate to derive:

\[
S_V(\omega) = \left| H^{(+)}(\omega_0) \right|^2 S
\]

\[
= \left| H^{(+)}(\omega_0) \right|^2 S \text{ for } \omega < 2\pi/T
\]

Comparing equations (18) and (19) we conclude that, in the switched capacitor circuit, it appears an excess noise with respect to the continuous circuit, being the output noise multiplied by the factor 1 + N(\omega).

In our case, from (17) the excess factor is:

\[
1 + N(\omega) = 1 + \sum_{k=0}^{\infty} \left| \frac{A(\omega_0 + k\omega_0)}{A(\omega_0 + k\omega_0)^2 + \gamma} \right|^2
\]
and for $\omega = 0$, we have:

\[
1 + N(0) = \frac{A_0}{B_0} \cosh \frac{A_0}{B_0}
\]

The result, expressed by (20) holds also for $\omega \ll \omega_T$. This excess noise factor is reported in Fig. 10 as a function of $2\gamma T \Lambda_0$. Fig. 10 points out that also to obtain low noise as large integrating bandwidth, it is requested to operate at largest possible sampling frequency $(1/T)$, practically around the gain-bandwidth frequency. For instance, in the particular case $2\gamma T = 1$ (for $\gamma = 1$) that is, if the sampling frequency $1/T$ is $\pi$ times larger than the unity gain frequency $A_0/2\pi$ of the operational amplifier, we get $1 + N(0) = 1.31$.

![Fig. 10](image)

**Fig. 10** - The noise d.c. excess factor as a function of the sampling frequency $1/T$

In Fig. 11 the output noise voltage spectra for different sampling frequencies are shown. In the same figure, the corresponding experimental points are reported. Also in the same figure, the dashed line refers to the RC continuous circuit, while the solid lines refer to the RC circuit, as calculated through (16), with $C_1/C = 0.5; 0.2; 0.1; 0.05$ (starting from the upper curve). The corresponding sampling frequencies are also indicated to maintain the same RC product.

The experimental measurements have been carried out again with the µA 741 operational amplifier (for which we measured $S^2 = 23$ mV/Hz$^{1/2}$), $C_1 = 100$ and $C$ of the values requested for $\gamma$.

In all the SC cases the experimental points rise up about the half of the sampling frequencies (corresponding to the switches clock frequency) for the clocking feed-through. Examining the curves of Fig. 11, we observe, as expected, that for the same RC integrating constant, the excess noise is lower by operating at higher clock frequency, accordingly to (20).

![Fig. 11](image)

**Fig. 11** - The normalized output noise spectra for RC = 15.6µs. The dashed line refers to the RC case, the solid lines refer to different SC realization with the same RC product, as calculated by using (16). Points refer to the corresponding observed measurements.

The noise measurements were carried out with a lock in amplifier PAR mod. 124 A used as selective r.m.s. meter.

5. CONCLUSIONS

For a switched capacitor active integrator the output voltage and noise depends also on the properties of the operational amplifier, mainly on the finite dc gain and the gain-bandwidth product.

The finite dc gain of the operational amplifier introduces the same low-frequency limit, in the continuous and sampled cases, for the integrating bandwidth. The upper frequency limit, instead, is normally given, in the SC case, by the clocking frequency and approaches the gain-bandwidth value, which is the limit in the continuous case, if the clocking frequency is made larger than that of the unity gain one.

The importance of an high clocking frequency is evident in the noise behaviour, being the output baseband noise affected by this frequency, through the aliasing effect. In fact, for clocking frequency $1/2T$ much lower than the gain-bandwidth value $A_0/2\pi$, the output noise arising from the internal noise voltage of the operational amplifier, is augmented by a factor $A_0/2\pi$; the noises in the continuous and sampled cases are almost equal only if the clocking frequency can be done larger than the gain-bandwidth.

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APPENDIX

The output voltage $V_0$ can be obtained from the circuit of Fig. 1A, where:

$$I(j\omega) = \frac{2C_1}{j\omega} \left[ \frac{1}{j\omega} \right] \left[ V_i^*(j\omega) - V_i(j\omega) \right]$$

is the Fourier transform of the (4), with $V_i = V_i^+$ and $V_i = V_i^-$. The star operator $*$ is defined as

$$V^*(j\omega) = \sum_{k=-\infty}^{\infty} V(t+k/T) e^{-j2\pi kn/T}$$

By inspection of the circuit, we can write:

$$\begin{cases}
V_0 = A(t) V_i^+ - V_i^-
\end{cases}$$

(1A)

$$jC_1(V_o - V_i) = -1 + jC_1(V_1 - V_i^-)$$

From (1A) we obtain for $V_1$:

$$V_1 = \frac{A}{A+y} V_i^+ + \frac{1}{A+y} V_i^- - D(V_i^+ - V_i^-)^*$$

(2A)

where

$$\gamma = 1 + C_1/C$$

and

$$D = \frac{2}{\omega \gamma} \frac{\gamma - 1}{A + \gamma}$$

By applying the star operator $*$ and remembering that $[D] = 1 + C_1/C$ we have from (2A):

$$V_1^* = \frac{1}{A+y} V_i^* + \frac{1}{A+y} V_i^-* + D^* [V_i^+ - V_i^-]^*$$

(3A)

Now, by substituting (3A) into (2A) and (2A) into the first equation of (1A), we get:

$$V_0 = \frac{A}{A+y} V_i^+ - (\gamma-1) V_i^- + B \left[ \frac{A}{A+y} V_i^+ + \frac{1}{A+y} V_i^- \right] - D^* [V_i^+ - V_i^-]^*$$

(4A)

being

$$B = \frac{D(A+y)}{1 + D^*} = \frac{2}{\omega \gamma} \frac{\gamma - 1}{1 + \frac{\gamma - 1}{A + \gamma}}$$

If we assume for $A$:

$$A = \frac{A_0}{1 - \omega T}$$

we calculate the explicit expression of $D^*$; it results:

$$D^* = \frac{2}{\omega \gamma} \frac{\gamma - 1}{A_0 - T} \frac{1 + \omega \gamma}{\omega \gamma + 1}$$

with:

$$v_0 = \frac{1}{A_0 - T}$$

taking the inverse Fourier transform of (4A) we get:

$$d(t) = \frac{1}{\omega \gamma} \left[ \frac{2(\gamma-1)}{A_0 - T} \right] e^{-t/\gamma} - \frac{1}{\omega \gamma} \left[ u(t) + \frac{A_0 - T}{\gamma} e^{-t/\gamma} u(t) \right]$$

Being $u(t)$ the unit step function, defined as:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Applying the star operator $*$ to (6A) and taking the Fourier transform, we obtain the expression of $D^*$ for $\gamma$:

$$D^* = \frac{2}{\omega \gamma} \frac{\gamma - 1}{A_0 - T} \left[ e^{\gamma(\gamma-1)} \right] + \frac{1}{\omega \gamma} \left[ \frac{A_0 - T}{\gamma} e^{-t/\gamma} u(t) \right]$$

(7A)